TITLE: SALARIES, INTEREST, AND INFLATION

Student/Class Goal
Because of inflation and interest rates, money often grows at an exponential rate. Students want to know how this affects them both positively and negatively.

Outcome (*lesson objective)*
Students will choose between the mean, median, and mode in financial situations, and then will justify their decision. Students will compare options based on the exponential growth of money.

Time Frame

2 hours

Standard *Use Math to Solve Problems and Communicate*

NRS EFL 5-6

Number Sense	Benchmarks	Geometry & Measurement	Benchmarks	Processes	Benchmarks
Words to numbers connection	5.1, 6.1	Geometric figures		Word problems	5.25, 6.26
Calculation	5.1	Coordinate system	5.7, 6.7	Problem solving strategies	5.26
Order of operations	5.2	Perimeter/area/volume formulas		Solutions analysis	5.27, 6.28
Compare/order numbers		Graphing two-dimensional figures		Calculator	5.28, 6.29
Estimation	5.4, 6.4	Measurement relationships		Math terminology/symbols	
Exponents/radical expressions	**5.5, 6.5**	Pythagorean theorem		Logical progression	
Algebra & Patterns	**Benchmarks**	Measurement applications		Contextual situations	**5.31, 6.32**
Patterns/sequences	5.15, **6.15**	Measurement conversions	5.13, 6.13	Mathematical material	5.32, 6.33
Equations/expressions	**5.16, 6.16**	Rounding	5.14, 6.14	Logical terms	
Linear/nonlinear representations	**6.17**	**Data Analysis & Probability**	**Benchmarks**	Accuracy/precision	
Graphing	5.18, 6.18	Data interpretation		Real-life applications	**5.35, 6.36**
Linear equations		Data displays construction		Independence/range/fluency	5.36, 6.37
Quadratic equations		Central tendency	**5.22, 6.23**		
		Probabilities			
		Contextual probability			

Materials
Interlocking Cubes
Calculators (if none are available, you can leave exponential equations in unreduced form)
Expected Salaries Task Sheet
Paying Bills and Inflation Task Handout
Savings Account Task Handout
Teacher Answer Sheet
Vocabulary Sheet

Learner Prior Knowledge

Students should be able to perform accurate calculations for exponential equations using order of operations. Students should be able to plot a coordinate pair on the X-Y plane, and then **interpolate** between points.

Instructional Activities

Step 1: Review. Write the following numbers on the board: 1, 4, 4, 3, 3, 7, 5, 4, 5. Ask students to write down the **mean**, **median**, **mode**, **range**, **five-number summary**, and **interquartile range (IQR)** of this set. If there are any terms that the students are not familiar with, give them the definition from the *Vocabulary Sheet*.

Step 2: Pass out 50 interlocking cubes to each student. To get them familiar with using the cubes, ask them to model the measures of center from Step 1. In other words, how could they use the cubes to figure out the mean, median, and mode of the set? There are two main types of methods the students might use (see *Teacher Answer Sheet*, Method 1 and Method 2); make sure they see them both (if they only come up with one, show them the other one also). Method 1: Each cube represents 1 unit.

Using this method, the students will actually need 36 cubes to model the problem. To find the median and mode, they must first arrange their stacks in order of height. To find them mean, take cubes from the tallest stacks and place them onto the smaller stacks until all the stacks are as equal as possible (for this problem, they will all equal 4). Method 2: Each cube represents 1 number. In this case, they will only need 9 cubes, but they will need some way to hold a place for each value, even the missing values of 2 and 6. Using this method, the mode is the highest stack, the median is the 5th cube from the left, and they can find the mean by taking a cube from any two stacks and placing both cubes in the middle. For example, they could take one cube from the 7 stack, and one cube from the 3 stack, and place them both on 5. Continuing in this manner, all cubes will eventually end up on 4, which is the mean.

Step 3: (I do) *Teacher models the solution process.* Pose this problem to students, "Change the original set of numbers, so that (a) the median stays the same, (b) the mode stays the same, and (c) the mean decreases." Use the talk aloud technique as you decide which method you want to use (either one is fine), and work your way through the problem. You may want to start with the mode and ask yourself out loud what it means to "keep the mode the same" (in other words, you can't change any of the 4's). When you feel you have an answer, make sure you go back through and check whether the median, mode, and mean of your modified set meet the requirements of the problem. Be sure to comment that this is not the only correct solution.

Step 4: (we do) *Teacher and students collaboratively work through the problem.* Pose this problem to students, "Change the original set of numbers, so that (a) the median decreases, (b) the mode stays the same, and (c) the mean increases." Allow students to choose a starting point for the problem, but if they struggle to decide, suggest that they begin with the mode again. The key learning point in Step 4 is that the mean is affected by how far each block is from the center, whereas the median is only impacted by whether a block is to the right or the left of the center.

Step 5: (you do) *Students independently work through the problem.* Pose this problem to students, "Find a set of five numbers for which (a) the median is 3, (b) the mode is 3, and (c) the mean is 5." Make sure that students check their answer against all three criteria when they finish. After everyone is done, have students share with the group how they approached the problem.

Step 6: (you do) *Students independently work through the problem.* Pose this activity to students, "Start with the set { 1, 1, 2, 3, 8 }. Find the mean, median, and mode." (pause, and allow students to work this out; compare answers) "Now, double every number in the set and find the new mean, median, and mode." (pause, work, compare) "Then, double every number in the new set and find the new mean, median, and mode." (pause, work, compare) "Based on these findings, make a **conjecture** about how doubling any set affects the mean, median, and mode of the set." Make sure students come up with the rule that doubling each number of a set also doubles the mean, the median, and the mode. Then tell students that this is true for any **exponential** increase or decrease (e.g., multiplying each number by 1.4 causes the mean, median, and mode to increase by 1.4 and multiplying each number by .5 causes the mean, median, and mode to be split in half).

Step 7: *Introduce the context*. Tell students that the three tasks will deal with common financial scenarios where measures of center are useful. However, as the tasks will show, it is important to understand the differences between the measures of center in order to interpret situations accurately. Another important concept in financial matters is exponential growth; two major examples are inflation and interest rates. Since growth is often constant, we will use the formula for continuous compounding in both of these types of situations.

$$\boxed{A = Pe^{rt}}$$

where t = the amount of time, r = the base rate of increase per 1 unit of time, P = principal (the initial amount of money), A = Amount (the final amount of money after *t* units of time), and e = Euler's number (2.7182…). Another related formula is the one for calculating **annual percentage yield** (APY).

$$\boxed{APY = e^{r} - 1}$$

Many banks and loan organizations publish both the base interest rate and the APY. The APY is the actual interest rate when the base rate is compounded throughout the period of time.

Step 8: (I do) *Teacher models the solution process through the Expected Salaries task.* Except for Question #6, these are basic calculation questions and conceptual questions. Although you can discuss the conceptual ones with students to some extent, remember that you are the leader for this step, and so you should be the one doing most of the talking as work through these problems. Before you do the calculations for Question #6, make a prediction out loud, based on the relationship you uncovered in Step 6 and the value for the range that you found in Question #1.

Step 9: (we do) *Teacher and students collaboratively work through the Paying Bills and Inflation task.* Allow students to take the initiative on the first question, but correct them immediately if you see them making a mistake. On Question #2 and #3, ask if anyone has any ideas before you give an explanation; make sure they justify their suggestions instead of just guessing wildly. If students seem completely puzzled, have them get out the interlocking cubes and work through a much simpler version of the same problem (for example, model a set of {2, 6, 7}, and then ask if it would be better to pay $2, $6, and $7 or the mean of $5 three times? Make sure that you choose a set where the mean and median are not equal, or else the students may get a false idea for Question #3). When you plot the graph in #4, make sure that the exponential curve pattern is clear (i.e., the relationship is not a straight line).

Step 10: (you do) *Students independently work through the Savings Account task.* Depending on your class dynamics, either partner students together or have them work individually. Before you pass out the task, explain that you want the students to tackle this problem as independently as possible. After passing out the handouts, walk around the room silently monitoring the students' progress. When you see them run into difficulties, try not to answer their questions directly; instead, remind them of similar situations from the first two tasks. On #4, make sure they don't try to solve for an exact answer, as this is a very complicated math problem. Instead, they can just look at where the lines on their graph intersect.

Step 11: Have each student (or pair) share both the process they used and their final comparisons. When students disagree, do not immediately provide the correct answer; allow each student or pair to try to convince the other first.

Assessment/Evidence (*based on outcome)*

Steps 9 and 10 will serve as evidence of student mastery. During Step 10, the teacher should actively listen to partner discussions for signs of understanding or of misconceptions. If students are working alone, the teacher should have students speak out loud as they solve the problem. During Step 11, allow students the opportunity to modify their solutions based on what they learn from watching others present their solutions.

Exit Slip: 1. Using interlocking cubes, find the mean of the following set of numbers: {3, 2, 3, 6, 7, 3, 4}
2. If you invest $500 in a 5-year Certificate of Deposit with a base rate of 5%, how much will you receive at the end of the five years?

Teacher Reflection/Lesson Evaluation

Not yet completed.

Next Steps

Have students take a look at either loans they are paying on, or savings they are accruing. They should find out the base interest rate (or APY) and calculate how much the interest will add up to over the span of 10 or 20 years.

Technology Integration

The site below offers explanations and actual figures for inflation rates over time. It also builds on inflation data with other financial concepts and data for students who are interested in exploring the topic further.
http://inflationdata.com/

The National Library of Virtual Manipulatives provides many exploratory tools. On this page, students can explore the "Loan Calculator" and the "Savings Calculator," for a more in-depth look at this lesson's topics.
http://nlvm.usu.edu/en/nav/category_g_4_t_5.html

Purposeful/Transparent

The fact that the value of money can change over time is both intriguing and confusing to many students. This lesson starts with simple visualizations (interlocking cubes), and progresses on to critical everyday situations such as salaries, bills, interest, and inflation.

Contextual

This lesson centers on the concept that the value of money is constantly changing – both the money we have and the money we owe. For any member of society (presumably all the ABLE learners), this is an important topic for gaining financial understanding and, ultimately, control.

Building Expertise

Students will build on their simple understanding of *calculating* the mean, median, and mode to understanding how each of them are affected by individual data points. Moreover, they will learn to make decisions based on this understanding of central tendency. Using interlocking cubes and their understanding of central tendency, they move on to realize the power of exponential functions over time.

Vocabulary Sheet

Annual Percentage Yield – the effective interest rate that accounts for compounding over the course of a year.

Conjecture – an educated guess based on incomplete evidence.

Exponential – numbers or quantities that are raised to an exponent.

Five Number-Summary – Q0, Q1, Q2, Q3, and Q4

Interpolate – estimate the y-values for all x's in between the minimum and maximum x-values.

Interquartile Range (IQR) – Q3 minus Q1

Mean – the sum of a set of values, divided by the number of elements in the set.

Median – the middle number in an ordered set (if the number of elements is odd) or the mean of the two middle numbers of an ordered set (if the number of elements is even).

Mode – the element(s) in a set which occur with the highest frequency.

Quartiles – Q0 is the smallest number in a set, Q2 is the median of the set, and Q4 is the largest number in the set. Q1 is the median of all the numbers before Q2 (and not including Q2) in the ordered set. Q3 is the median of all the numbers after Q2 (and not including Q2) in the ordered set.

Range – Q4 minus Q0